Universal Approach to Overcoming Nonstationarity, Unsteadiness and Non-Markovity of Stochastic Processes in Complex Systems

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In present paper we suggest a new universal approach to study complex systems by microscopic, mesoscopic and macroscopic methods. We discuss new possibilities of extracting information on nonstationarity, unsteadiness and non-Markovity of discrete stochastic processes in complex systems. We consider statistical properties of the fast, intermediate and slow components of the investigated processes in complex systems within the framework of microscopic, mesoscopic and macroscopic approaches separately. Among them theoretical analysis is carried out by means of local noisy time-dependent parameters and the conception of a quasi-Brownian particle (QBP) (mesoscopic approach) as well as the use of wavelet transformation of the initial row time series. As a concrete example we examine the seismic time series data for strong and weak earthquakes in Turkey (1998, 1999) in detail, as well as technogenic explosions. We propose a new way of possible solution to the problem of forecasting strong earthquakes forecasting. Besides we have found out that an unexpected restoration of the first two local noisy parameters in weak earthquakes and technogenic explosions is determined by exponential law. In this paper we have also carried out the comparison and have discussed the received time dependence of the local parameters for various seismic phenomena.

I. INTRODUCTION

Nonstationarity, unsteadiness and non-Markovity are the most common essential peculiarities of stochastic processes in nature. The existence of the similar properties creates significant difficulties for the theoretical analysis of real complex systems [1]. At present, methods, connected with localization of registered or calculated parameters for the quantitative account of the dramatic changes caused by the fast alternation of the behavior modes and intermittency, came into use. For example, the time behavior of the local (scale) Hurst exponents was defined in the recent work of Stanley *et al.* to study multifractal cascades in heartbeat dynamics [1] and to analyze and forecast earthquakes and technogenic explosions in Ref. [2]. The application of the local characteristics allows one to avoid difficulties connected with nonergodicity of the investigated system and gives a possibility to extract additional valuable information on the hidden properties of real complex systems. From the physical point of view, this approach resembles the use of nonlinear equations of generalized hydrodynamics with the local time behavior of hydrodynamical and thermodynamical parameters and characteristics.

It is well known that one of the major problems of seismology is to predict the beginning of the main shock. Though science seems still to be far from the guaranteed decision of this problem there exist some interesting approaches based on the peculiar properties of precursory phenomena [3, 4, 5, 6, 7, 8, 9]. Another important problem is recognition and differentiation of weak earthquakes and technogenic underground explosions signals. One of useful means of solving this problem is the defining of corresponding local characteristics [1, 2].

In the present work we suggest a new universal description of real complex systems by means of the microscopic, mesoscopic and macroscopic methods. We start with a macroscopic approach based on the kinetic theory of discrete stochastic processes and the hierarchy of the chain of finite-difference kinetic equations for the discrete time correlation function (TCF) and memory functions [2, 11, 12].

The mesoscopic phenomena of the so-called "soft matter" physics, embracing a diverse range of system including liquid crystals, colloids, and biomembranes, generally involve some form of coupling of different characteristic time- and length-scales. Computational modelling of such multi-scale effects requires a new methodology applicable beyond the realm of traditional techniques such as *ab initio* and classical molecular dynamics (the methods of choice in the microscopic regime), as well as phase field modelling or the lattice-Boltzmann method (usually concerned with the macroscopic regime). As for complex systems, we propose to consider intermediate and slow processes within a unified framework of mesoscopic approach: by means of local time behavior of the local relaxation and kinetic parameters, local non-Markovity parameters and so on. For this purpose we introduce the notion of quasi-Brownian motion in a complex system by coarse-grained averaging of the initial time series on the basis of wavelet transformation.

As an example we consider here the local properties of relaxation or noise parameters for the analysis of seismic phenomena such as earthquakes and technogenic explosions. The layout of the paper is as follows. In Sec. II we describe in brief the stochastic dynamics of time correlation in complex systems by means of discrete non-Markov kinetic equations. Basic equations used for these calculations are presented here. The local noise parameters are defined in Sec. III. Section IV contains results obtained by the local noise parameter procedure for the case of seismic signals. The models of the time dependence of the local parameters are given in Sec. V. The basic conclusions are discussed in Sec. VI.

II. THE BASIC DEFINITIONS IN KINETIC DESCRIPTION OF DISCRETE STOCHASTIC PROCESSES

A. MACROSCOPIC DESCRIPTION IN THE ANALYSIS OF STOCHASTIC PROCESSES

A lot of different existent processes, such as economical, metheorogical, gravimetrical and other, are registered as discrete random series x_i of some variable X. This random variable X can be written as an array of its values

$$X = \{x(T),\ x(T+\tau),\ x(T+2\tau),\ ...,\ x(T+k\tau),\ ...,\ x(T+(N-1)\tau)\}. \tag{1}$$

Here a time step (or a time interval) τ is a constant, T is the time when the registration of the signal begins, $(N-1)\tau$ is the duration of the signal detection.

The average value $\langle x\rangle$ and fluctuations δx_j are defined by the following expressions, correspondingly

$$\langle x\rangle = \frac{1}{N}\sum_{j=0}^{N-1} x(T+j\tau), \quad \delta x_j = \delta x(T+j\tau) = x(T+j\tau) - \langle x\rangle. \tag{2}$$

From the fluctuations of the considered random variable δx_j we can form k-component state vector of the system's correlation state

$$\mathbf{A}_k^0 = \mathbf{A}_k^0(0) = (\delta x(T),\ \ \delta x(T+\tau),\ \ ...,\ \ \delta x(T+(k-1)\tau)) = (\delta x_0,\ \ \delta x_1,\ \ ...,\ \ \delta x_{k-1}). \tag{3}$$

The time dependence of the correlation state vector $\mathbf{A}$ can be represented as a discrete m-step shift

$$\mathbf{A}_{m+k}^m = \mathbf{A}_{m+k}^m(t) = (\delta x(T+m\tau),\ \ \delta x(T+(m+1)\tau),\ \ ...,\ \ \delta x(T+(m+k+1)\tau)) = (\delta x_0,\ \ \delta x_1,\ \ ...,\ \ \delta x_{m+k-1}). \tag{4}$$

Then by analogy with the papers [2, 11, 12] we can write the following normalized TCF

$$M_0(t) = \frac{\langle \mathbf{A}_{N-1-m}^0 \mathbf{A}_{N-1}^m\rangle}{\langle \mathbf{A}_{N-1-m}^0 \mathbf{A}_{N-1-m}^0\rangle} = \frac{\langle \mathbf{A}_{N-1-m}^0(0) \mathbf{A}_{N-1}^m(t)\rangle}{|\mathbf{A}_{N-1-m}^0(0)|^2} = \frac{\langle \mathbf{A}_{N-1-m}^0(0) U(t=T+m\tau,T) \mathbf{A}_{N-1-m}^0(0)\rangle}{|\mathbf{A}_{N-1-m}^0(0)|^2}, \tag{5}$$

where angular brackets indicate the scalar product of the two state vectors. On the other hand, the time dependence of the vector $\mathbf{A}_{N-1-m}^m(T+t)$, $t=m\tau$, can be represented formally with the help of the evolution operator $U(t',t)$ as follows:

$$\mathbf{A}_{N-1}^m(T+t) = U(T+m\tau,T)\mathbf{A}_{N-1-m}^0(T) = U(t,0)\mathbf{A}_{N-1-m}^0(0). \tag{6}$$

The last one has the property: $U(t,t)=1$. Actually, one can write down formal discrete equation of motion with the use of the evolution operator $U(t',t)$ (see Appendix A for more details).

It was shown in Refs. [2, 11, 12] that the finite-difference kinetic equation of a non-Markov type for TCF $M_0(t)$ can be written by means of the technique of projection operators of Zwanzig'-Mori's type [15, 16] as

$$\frac{\triangle M_0(t)}{\triangle t} = \lambda_1 M_0(t) - \tau\Lambda_1 \sum_{j=0}^{m-1} M_1(j\tau) M_0(t-j\tau). \tag{7}$$

Here the first order memory function $M_1(j\tau)$ appears, λ_1 is the eigenvalue of Liouville's quasi-operator $\hat{\mathcal{L}}$ and Λ_1 is the relaxation noise parameter, which are characteristics of the investigated process. Possible methods of defining quasioperator $\hat{\mathcal{L}}$ are presented in Appendix A [see Eqs. (A3),(A5) and (A7)]. It should be recorded that Eq. (7) is the first kinetic finite-difference equation for initial TCF $M_0(t)$. With the use of the same procedure of projection operator we can obtain the chain of kinetic finite-difference equations of the following form:

$$\frac{\triangle M_{i-1}(t)}{\triangle t} = \lambda_i M_{i-1}(t) - \tau\Lambda_i \sum_{j=0}^{m-1} M_i(j\tau) M_{i-1}(t-j\tau), \quad i=1,2,3,.... \tag{8}$$

Here $M_i(j\tau)$ is the memory function of the ith order, whereas λ_i and Λ_i are noise parameters:

$$\lambda_n = i\frac{\langle \mathbf{W}_{n-1}\hat{\mathcal{L}}\mathbf{W}_{n-1}\rangle}{|\mathbf{W}_{n-1}|^2}; \quad \Lambda_n = i\frac{\langle \mathbf{W}_{n-1}\hat{\mathcal{L}}\mathbf{W}_{n}\rangle}{|\mathbf{W}_{n-1}|^2}. \tag{9}$$

Here $\mathbf{W}_n$ are the dynamical orthogonal variables, obtained by the Gram-Schmidt orthogonalization procedure

$$\langle \mathbf{W}_n, \mathbf{W}_m\rangle = \delta_{n,m}\langle|\mathbf{W}_n|^2\rangle,$$

where $\delta_{n,m}$ is the Kronecker's symbol,

$$\mathbf{W}_0 = \mathbf{A}_k^0(0), \quad \mathbf{W}_1 = [i\hat{\mathcal{L}} - \lambda_1]\mathbf{W}_0,$$

$$\mathbf{W}_2 = [i\hat{\mathcal{L}} - \lambda_2]\mathbf{W}_1 - \Lambda_1\mathbf{W}_0, ... \tag{10}$$

From Eq. (10) it is obvious that in the cited Gram-Schmidt procedure from each new vector of state one should subtract the projection on all the previous vectors. Thereafter the orthogonalization (10) is complete.

A chain of integro-differential equations (8) arise as a result of the use of projection operator technique to define different correlation functions in physical problems [15, 16] (for example, TCF of density fluctuation in Inelastic Neutron Scattering [13, 14] and Light Scattering investigations, velocity autocorrelation function and others can be received in the specified way). However, the quest for the physically based way of closing the chain of equations and finding the spectra of the initial TCF are essential moments in these challenges. Here the situation is different. Namely, the initial TCF can be calculated directly from the experimental data. Then memory functions $M_i(t)$ and noise parameters λ_i, Λ_i are similarly calculated from the experiment. All these functions and parameters make it possible to carry out the detailed analysis of the random process. The noise parameters λ_i and Λ_i are the relaxation characteristics of the experimental time series, which contain information of various modes passing and changing. The macroscopic approach presented above is based on the calculation of memory functions, power spectra, dynamic variables, relaxation parameters. It suggests the investigation of the system as a single whole. The global characteristics calculated on the basis of all the time series contain hidden information about various modes of the system behavior. As a rule, this information is difficult to extract and analyze. For this reason it is necessary to develop a mesoscopic description and introduce a local noise and relaxation parameters λ_i and Λ_i.

B. MESOSCOPIC DESCRIPTION IN ANALYSIS OF STOCHASTIC PROCESSES

It is well known, the mesoscopic conception is one of the possible ways to deal with random processes in complex systems. It consists of extension of the domain of the dynamical equations mesoscopic variables and of introduction of a some local time interval. Such quantity as X (it can be particle position, mass density and so on) is defined on the mesoscopic space [23]. In addition we introduce number M as its extensive quantity. This number M should satisfy the condition:

$$1 \ll M \ll N, \tag{11}$$

where N is the length of the initial experimental sampling. Let us take a working window of the fixed length M. By superposing this window on the initial sampling X, we choose all elements, incoming into it, as a separate sampling $\bar{\xi}_0$. Further, let us execute one time step τ shift of this working window to the right and obtain another local sampling of the length M. Executing this procedure $(N-M+1)$ times, one can obtain the same quantity of the local samplings of length M:

$$\begin{aligned}
\bar{\xi}_0 &= \bar{\xi}_0\{x(T), x(T+\tau), x(T+2\tau), \dots, x(T+(M-1)\tau)\}, \\
\bar{\xi}_1 &= \bar{\xi}_1\{x(T+\tau), x(T+2\tau), x(T+3\tau), \dots, x(T+M\tau)\}, \\
& \dots, \\
\bar{\xi}_{N-M} &= \bar{\xi}_{N-M}\{x(T+(N-M-1)\tau), x(T+(N-M)\tau), x(T+(N-M+1)\tau) \dots, x(T+(N-1)\tau)\}. \quad (12)
\end{aligned}$$

The obtained local samplings $\bar{\xi}_i$ form the array, which represents the time dynamics of the investigated process,

$$\bar{\xi}(t') = \{\bar{\xi}_0, \bar{\xi}_1, \bar{\xi}_2, \dots, \bar{\xi}_i, \dots, \bar{\xi}_{N-M}\}. \quad (13)$$

Then, in accordance with the procedure described in the last subsection (II A), we can define fluctuations by Eq. (2), calculate TCF with the help of Eq. (5), calculate memory functions and parameters λ_i and Λ_i by Eqs. (9) for every sampling $\bar{\xi}_i$. However, parameters λ_i and Λ_i will be characteristics of concrete jth sampling only. To characterize the local properties of the initial time data X, it is convenient to represent their local time dependence in the following way:

$$\begin{aligned}
\lambda_i(t') &= \{\lambda_i(T+(M-1)\tau), \lambda_i(T+M\tau), \lambda_i(T+(M+1)\tau), \dots, \lambda_i(T+(N-M)\tau)\}, \\
\Lambda_i(t') &= \{\Lambda_i(T+(M-1)\tau), \Lambda_i(T+M\tau), \Lambda_i(T+(M+1)\tau), \dots, \Lambda_i(T+(N-M)\tau)\}. \quad (14)
\end{aligned}$$

Operating in the similar way, one can execute cross-over from the macroscopic description of the whole system to the mesoscopic one. The offered approach is very convenient for the description and analysis of non-stationary stochastic processes. It allows to depart from the global macro-characteristics, which carry only averaged minor information about the whole investigated process, and to turn to the stochastic description with the use of the local characteristics, and, as a result, to execute a more detailed analysis of various dynamic states of the system.

C. CONCEPTION OF ONE-DIMENSIONAL DYNAMICS OF QUASI-BROWNIAN PARTICLE

Let us consider the motion of a large Brownian particle in a dense medium composed of light molecules and restrict it by a simple one-dimensional case. The coordinate x_i and the velocity v_i are random variables of a Brownian particle. The quantity τ represents average time between the two successive collisions of liquid molecules, T is the initial moment of time. the variable M characterizes a local size (mass) of a Brownian particle. It is obvious, that a Brownian particle must have a larger mass in comparison with liquid particles ($M \gg 1$), therefore it is more inert. Then it is convenient to define the coordinate of a Brownian particle at moment t' as an average value of sampling $\bar{\xi}_i$. For example, we obtain from $\bar{\xi}_0$:

$$y_0 = \frac{x(T) + x(T+\tau) + x(T+2\tau) + \dots + x(T+(M-1)\tau)}{M} = \frac{1}{M}\sum_{j=0}^{M-1} x(T+j\tau). \quad (15)$$

The quantity y_0 defines the coordinate of "the center of mass" of a Brownian particle at the initial time moment $t' = 0$. By analogy, it can define the position of a Brownian particle at the next time moment $t' = \tau$ and so on. As a result, we obtain a new time discrete series $Y(t')$ as:

$$Y(t') = \{y_0, y_1, y_2, \dots, y_{N-M}\}. \tag{16}$$

The velocity of a Brownian particle is

$$v_i = \frac{y_{i+1} - y_i}{\tau}. \tag{17}$$

So, for example, for the initial velocity v_0 we obtain from Eqs. (12), (15) and (17):

$$v_0 = \frac{y_1 - y_0}{\tau} = \frac{x(T + M\tau) - x(T)}{M\tau}. \tag{18}$$

Obviously, *if the larger one is M, the smaller one is the velocity of a Brownian particle.* So, we obtain a discrete set of velocity values for a Brownian particle at equally small time interval τ

$$V(t') = \{v_0, v_1, v_2, \dots, v_{N-M-1}\}. \tag{19}$$

Eqs. (16) and (19) define time dependence of random variables $Y(t')$ and $V(t')$.

1. Generalized Langevin equation with discrete time

In accordance with Eq. (2) we define the correlation state vector, components of which are the fluctuations of the particle position,

$$\mathbf{B}_k^0 = \{\delta y_0,\ \delta y_1,\ \delta y_3, \dots,\ \delta y_{k-1}\}. \tag{20}$$

Then, by analogy to Eq. (4) time dependence of the vector $\mathbf{B}$ can be considered a discrete m-step time shift. For the vector $\mathbf{B}$ the following normalized TCF can be written with the help of Eq. (5)

$$b(t) = \frac{\langle \mathbf{B}_{N-1-m}^0 \mathbf{B}_{N-1}^m \rangle}{|\mathbf{B}_{N-1-m}^0|^2}. \tag{21}$$

The last one describes the time correlation of the two different correlation states of the system.

Now we introduce the linear projection operator in Euclidean space of the state vectors

$$Q\mathbf{B} = \frac{\mathbf{B}(0)\rangle\langle\mathbf{B}(0)\mathbf{B}(t)\rangle}{|\mathbf{B}(0)|^2} = \mathbf{B}(0)\rangle\ b(t),\ \ Q = \frac{\mathbf{B}(0)\rangle\langle\mathbf{B}(0)}{\langle\mathbf{B}(0)\mathbf{B}(0)\rangle}. \tag{22}$$

This operator has the necessary property of idempotency $Q^2 = Q$. The existence of projection operator Q allows to introduce the mutually-supplementary projection operator R as follows:

$$R = 1 - Q,\ \ R^2 = R,\ \ QR = RQ = 0. \tag{23}$$

It is necessary to note that the projectors Q and R are both linear and can be recorded for the fulfillment of projection operations in the particular Euclidean space. The projection operators Q and R allow one to carry out the splitting of Euclidian space of vectors B, where $B(0)$ and $B(t) \in B$, into a straight sum of the two mutually supplementary subspaces in the following way:

$$B = B' + B'',\ \ B' = QB,\ \ B'' = RB. \tag{24}$$

Let us consider the finite-difference Liouville's Eq. (A1) for the vector of fluctuations of a Brownian particle position

$$\frac{\triangle}{\triangle t}\mathbf{B}^m_{m+k}(t) = i\hat{\mathcal{L}}(t,\tau)\mathbf{B}^m_{m+k}(t). \tag{25}$$

Affecting the last equation by the operators Q and R successfully, we can split the dynamic equation (25) into two interconnected equations in the two mutually-supplementary Euclidean subspaces:

$$\frac{\triangle}{\triangle t}\mathbf{B}'(t) = iQ\hat{\mathcal{L}}(Q+R)\mathbf{B}(t) = i\hat{\mathcal{L}}_{11}\mathbf{B}'(t) + i\hat{\mathcal{L}}_{12}\mathbf{B}''(t), \tag{26}$$

$$\frac{\triangle}{\triangle t}\mathbf{B}''(t) = iR\hat{\mathcal{L}}(Q+R)\mathbf{B}(t) = i\hat{\mathcal{L}}_{21}\mathbf{B}'(t) + i\hat{\mathcal{L}}_{22}\mathbf{B}''(t). \tag{27}$$

In the above equations the matrix elements $\hat{\mathcal{L}}_{ij}$ of the quasi-operator $\hat{\mathcal{L}}$ have been introduced

$$\hat{\mathcal{L}}_{11} = Q\hat{\mathcal{L}}Q,\ \hat{\mathcal{L}}_{12} = Q\hat{\mathcal{L}}R,\ \hat{\mathcal{L}}_{21} = R\hat{\mathcal{L}}Q,\ \hat{\mathcal{L}}_{22} = R\hat{\mathcal{L}}R,$$

$$\hat{\mathcal{L}} = \begin{pmatrix} \hat{\mathcal{L}}_{11} & \hat{\mathcal{L}}_{12} \\ \hat{\mathcal{L}}_{21} & \hat{\mathcal{L}}_{22} \end{pmatrix}. \tag{28}$$

Operators $\hat{\mathcal{L}}_{ij}$ act as follows: $\hat{\mathcal{L}}_{11}$ - from a subspaces B' to B'; $\hat{\mathcal{L}}_{12}$ - from B'' to B'; $\hat{\mathcal{L}}_{21}$ - from B' to B''; and $\hat{\mathcal{L}}_{22}$ - from B'' to B''.

To simplify the Liouville Eqs. (26) and (27), we exclude the irrelevant part $B''(t)$ and construct the closed equation for the relevant part $B'(t)$. For this purpose, it is necessary to obtain a step-by-step solution of Eq. (27)

$$\frac{\triangle \mathbf{B}''(t)}{\triangle t} = \frac{\mathbf{B}''(t+\tau) - \mathbf{B}''(t)}{\tau} = i\hat{\mathcal{L}}_{21}\mathbf{B}'(t) + i\hat{\mathcal{L}}_{22}\mathbf{B}''(t), \tag{29}$$

$$\begin{aligned}\mathbf{B}''(t+\tau) &= \mathbf{B}''(t) + i\tau\hat{\mathcal{L}}_{21}\mathbf{B}'(t) + i\tau\hat{\mathcal{L}}_{22}\mathbf{B}''(t) \\ &= (1 + i\tau\hat{\mathcal{L}}_{22})\mathbf{B}''(t) + i\tau\hat{\mathcal{L}}_{21}\mathbf{B}'(t) \\ &= U_{22}(t+\tau,t)\mathbf{B}''(t) + i\tau\hat{\mathcal{L}}_{21}(t+\tau,t)\mathbf{B}'(t),\end{aligned} \tag{30}$$

where $U_{22}(t+\tau,t) = 1 + i\tau\hat{\mathcal{L}}_{22}(t+\tau,t)$ is the operator of a time step shift.

With the help of Eq. (30) we can derive the following expression for the next time step:

$$\begin{aligned}\mathbf{B}''(t+2\tau) &= U_{22}(t+2\tau,t+\tau)\mathbf{B}''(t+\tau) + i\tau\hat{\mathcal{L}}_{21}(t+2\tau,t+\tau)\mathbf{B}'(t+\tau) \\ &= U_{22}(t+2\tau,t+\tau)[U_{22}(t+\tau,t)\mathbf{B}''(t) + i\tau\hat{\mathcal{L}}_{21}(t+\tau,t)\mathbf{B}'(t)] \\ &\quad + i\tau\hat{\mathcal{L}}_{21}(t+2\tau,t+\tau)\mathbf{B}'(t+\tau) \\ &= U_{22}(t+2\tau,t+\tau)U_{22}(t+\tau,t)\mathbf{B}''(t) + i\tau[U_{22}(t+2\tau,t+\tau) \\ &\quad \times\hat{\mathcal{L}}_{21}(t+\tau,t)\mathbf{B}'(t) + \hat{\mathcal{L}}_{21}(t+2\tau,t+\tau)\mathbf{B}'(t+\tau)].\end{aligned} \tag{31}$$

Generalizing this result in case of the mth discrete step we find the following final result

$$\begin{aligned}\mathbf{B}''(t+m\tau) &= \left\{\hat{T}\prod_{j=0}^{m-1} U_{22}(t+(j+1)\tau, t+j\tau)\right\}\mathbf{B}''(t) \\ &\quad + i\tau\sum_{j=0}^{m-1}\left\{\hat{T}\prod_{j'=j}^{m-2} U_{22}(t+(j'+2)\tau, t+(j'+1)\tau)\right\} \\ &\quad \times\hat{\mathcal{L}}(t+(j+1)\tau, t+j\tau)\mathbf{B}'(t+j\tau).\end{aligned} \tag{32}$$

Here $\hat{T}$ denotes the Dyson operator of chronological ordering. Substituting the irrelevant part of Eq. (26) in the right side of Eq. (32), we obtain the closed finite-difference equation for the relevant part of the correlation state vector

$$\begin{aligned}\frac{\triangle}{\triangle t}\mathbf{B}'(t+m\tau) &= i\hat{\mathcal{L}}_{11}(t+(m+1)\tau, t+m\tau)\mathbf{B}'(t+m\tau) + i\hat{\mathcal{L}}_{12}(t+(m+1)\tau, t+m\tau) \\ &\times(\left\{\hat{T}\prod_{j=0}^{m-1}U_{22}(t+(j+1)\tau, t+j\tau)\right\}\mathbf{B}''(t) - \tau\sum_{j=0}^{m-1}\{\hat{T}\prod_{j'=j}^{m-2}U_{22}(t+(j'+2)\tau, t \\ &+(j'+1)\tau)\}\hat{\mathcal{L}}_{21}(t+(j+1)\tau, t+j\tau)\mathbf{B}'(t+j\tau)).\end{aligned} \tag{33}$$

Substituting Eqs. (22) and (24) in Eq. (33), we derive a finite-difference kinetic equation of a non-Markov type for TCF $b(t)$

$$\frac{\triangle b(t)}{\triangle t} = \lambda_1 b(t) - \tau\Lambda_1\sum_{j=0}^{m-1}M_1(t-j\tau)b(j\tau). \tag{34}$$

Here the TCF $M_1(t)$ is the first order memory function

$$M_1(t-j\tau) = \frac{\langle \mathbf{W}_1(0)\mathbf{W}_1(t-j\tau)\rangle}{|\mathbf{W}_1(0)|^2}, \tag{35}$$

Here Λ_1 is the frequency relaxation parameter of the first order with a square frequency dimension, and is defined by Eq. (9). Then if the dynamic variable $\mathbf{W}_0 = \mathbf{B}_k^0(0)$ represents the fluctuations of a Brownian particle position, the dynamic variable $\mathbf{W}_1 = i\hat{\mathcal{L}}\mathbf{W}_0 - \lambda_1\mathbf{W}_0$ contains fluctuations of pulses of a Brownian particle [see Eqs. (10)]. The function $M_1(t)$ is time correlation function of fluctuations of a Brownian particle velocity.

Defining the corresponding projection operators to new dynamic variable $\mathbf{W}_1$ and repeating the above described procedure, we find the finite-difference kinetic equation for $M_1(t)$

$$\frac{\triangle M_1(t)}{\triangle t} = \lambda_2 M_1(t) - \tau\Lambda_2\sum_{j=0}^{m-1}M_2(t-j\tau)M_1(j\tau). \tag{36}$$

Here λ_2 and Λ_2 are the frequency relaxation parameters of the second order, $M_2(t)$ is the second order memory function or memory function of the velocity correlation function for a Brownian particle (memory friction) [19, 20, 21, 22]. In fact, Eq. (36) is a discrete finite-difference *generalized Langevin equation* (GLE). So, $M_2(t)$ can be associated with TCF of the stochastic Langevin forces, for which the similar equation can be received

$$\frac{\triangle M_2(t)}{\triangle t} = \lambda_3 M_2(t) - \tau\Lambda_3\sum_{j=0}^{m-1}M_3(t-j\tau)M_2(j\tau) \tag{37}$$

with the third order frequency relaxation parameters λ_3 and Λ_3, and the memory function of the third order $M_3(t)$ respectively.

The three Eqs. (34), (36) and (37) are the exact consequence of microscopic discrete finite-difference equations of motion. Calculation of the memory function $M_i(t)$ and the relaxation frequency parameters λ_j, Λ_k are the central point here.

In case of a Brownian particle presented above, dynamic variables $\mathbf{W}_0$ and $\mathbf{W}_1$ are the position and pulses of random particles. In fact, they can be taken as characteristics of any other non-stationary process. The averaging operator of sampling with the length M, $\hat{A} = 1/M \sum_{j=0}^{M-1}$ [see Eq. (15)], applied to the intermediate local sampling, can be changed by any other operator, depending on the goal of the investigation. The operator $\hat{A}$ allows one to get clear of sharp fluctuations in the initial sampling of data $X(t)$ and to replace it by an other $Y(t')$: $X(t) \rightarrow Y(t')$, which contains the results of coarse-graining averaging. However, it is not always convenient to average a local sampling. In these cases the operator $\hat{A}$ can be replaced by an other operator, which allows one to obtain only one number from every local sampling. In general case, an another, a more universal method of transformation of the initial sampling into the sampling with some specified (required) characteristics can become discrete wavelet-transform [24, 25, 26, 27, 28], which is defined by the following equation:

$$W(j,k) = \sum_j \sum_k X(k) 2^{-j-2} \Psi(2^{-j} n - k). \tag{38}$$

Here $X(k)$ is the sampling (1) and $\Psi(t)$ is a time function with fast decay called mother wavelet. The following analysis can be applied to the new transformed data $W(j,k)$ according to the algorithm described above. Namely, memory functions $M_i(t)$ and frequency relaxation parameters λ_i and Λ_i can be calculated for the transformed data.

2. *Analogue of Green-Kubo relation for diffusion coefficient for time discrete system*

According to the theory of random walkers the mean-square displacements of a Brownian particle can be defined as

$$\langle \triangle y^2 \rangle_t = \int_{-\infty}^{+\infty} \triangle y^2 \Phi_1(y,t) dy = 2Dt, \qquad (t \rightarrow \infty). \tag{39}$$

Here $\Phi_1(y,t)$ is the density of probability of being of particle at point y at time moment t. The Eq. (39) is a well known Einstein relation for continuous displacements.

According to Eq. (39) the displacement during time t is

$$\triangle y_t = y(t) - y(0). \tag{40}$$

In case of the discrete time Eq. (40) can be rewritten in the form

$$\triangle y(t) = \triangle y(T + j\tau) = y(T + (j+m)\tau) - y(T + j\tau), \; t = m\tau, \qquad m \gg 1. \tag{41}$$

Then the mean-square displacement of a Brownian particle is

$$\langle \triangle y^2 (T + (j+m)\tau) \rangle = \frac{1}{N-m} \sum_{j=0}^{N-m-1} [y(T + (j+m)\tau) - y(T + j\tau)]^2, \qquad N > m \gg 1, \tag{42}$$

where $N - m$ is the quantity of "possible ways". The diffusion coefficient takes the following form

$$D = \frac{1}{2m(N-m)\tau} \sum_{j=0}^{N-m-1} [y(T + (j+m)\tau) - y(T + j\tau)]^2, \qquad \text{at } N > m \gg 1 \text{ (or } t \rightarrow \infty \text{)}. \tag{43}$$

Let's consider separately the sum in the last expression in terms of the velocity [see Eq. (17)]

$$\begin{aligned}
\sum_{j=0}^{N-m-1} [\ y\ \ & (T+(j+m)\tau)-y(T+j\tau)]^2 \\
&= \underbrace{[y(T+m\tau)-y(T)]^2+[y(T+(1+m)\tau)-y(T+\tau)]^2+\ldots+[y(T+(N-1)\tau)-y(T+(N-m-1)\tau)]^2}_{(N-m)\text{ of square brackets }[...]} \\
&= [y(T+m\tau)-y(T+(m-1)\tau)+y(T+(m-1)\tau)-...-y(T)]^2+\ldots \\
&\quad [y(T+(N-1)\tau)-y(T+(N-2)\tau)+y(T+(N-2)\tau)-...-y(T+(N-m-1)\tau)]^2 \\
&= \tau^2[v(T+(m-1)\tau)+v(T+(m-2)\tau)+...+v(T)]^2+\ldots \\
&+ \tau^2[v(T+(N-2)\tau)+v(T+(N-3)\tau)+\ldots+v(T+(N-m-1)\tau)]^2 \\
&= \tau^2\sum_{j=0}^{N-m-1}\left[\sum_{j=0}^{m-1+j} v(T+k\tau)\right]^2 . \qquad (44)
\end{aligned}$$

Then the expression for the diffusion coefficient (43) can be rewritten as:

$$D=\frac{\tau}{2m(N-m)}\sum_{j=0}^{N-m-1}\left[\sum_{k=j}^{m-1+j} v(T+k\tau)\right]^2 . \qquad (45)$$

Eq. (45) is the discrete finite-difference analog of the famous Green-Kubo relation for the diffusion coefficient. Given relation has been obtained from Einstein equation (39) for a discrete system. The asymptotic limit $t\to\infty$ can be replaced here by the similar condition $N\to\infty$, $m\to\infty$, $N>m$.

III. LOCAL NOISY PARAMETERS

Seismic data represent discrete random series, which is recording of displacements of the Earth's surface. Therefore, we can use the above-stated formalism to analyze seismic data. In particular, the local dependence of various characteristics [1, 2] can be serve as additional source of information on properties of objects. The noise parameters λ_i and Λ_i are very sensitive to the presence of a nonrandom component in the sampling. The change of the character of the correlated noise and, the appearance of the additional signal in the sampling can cause the alternation of these parameters. So, the time behavior of the local parameters λ_i, Λ_i is important and informative for analyzing of seismic data.

The procedure of localization consists in the following. Let us assume that we have an array of data $\{x_1,\ \ x_2,\ \ x_3,...,x_M,...,x_N\}$ and take the initial sampling of the fixed length M. Then by passing through all array of values with the "work window" of the fixed length M we can calculated the time series of the noisy parameters $\{\lambda_i(T,T+M\tau),\ \ \lambda_i(T+\tau,T+(M+1)\tau),...,\lambda_i(T+(N-M-1)\tau,T+(N-1)\tau)\}$ and $\{\Lambda_i(T,T+M\tau),\ \ \Lambda_i(T+\tau,T+(M+1)\tau),...,\Lambda_i(T+(N-M-1)\tau,T+(N-1)\tau)\}$

Obviously, it is inadmissible to use both very large intervals (M =1000 and more points) and very short intervals (M =50 points and less) for definition of local parameters $\lambda_i(t)$, $\Lambda_i(t)$. In the first case the physical sense of the localization procedure is lost. On the other hand, it is impossible to carry out any plausible correlation analysis with small intervals because of gross errors. Therefore there is necessity for finding the optimum length of the initial local interval or quantity M.

To determine the optimal minimal local sampling we have used the data corresponding to the calm state of the Earth before the technogenic explosion (an underground nuclear explosion). The calculation procedure consists in the following. We have taken the interval of 40 points as the starting point and have calculated all the low-order noise parameters λ_i $(i = 1, 2, 3)$ and Λ_j $(j = 1, 2)$ by Eqs. (9) and (10). Then the interval was consistently increased by unit time segment τ and the relaxation parameters λ_i, Λ_j were calculated every time at the increase of the interval. As a result of this procedure executed for the calm state of the Earth we have established that all parameters take "steady" numerical values at the interval approximately equal to 150 points and more (see Fig. 1 for more details). Namely, from Fig. 1 one can see, that the parameters λ_1 and Λ_1 take minimal values in their absolute quantity at this length of interval, the amplitude of value fluctuations of the parameter Λ_2 gets lower and level off. It is again the evidence of reduction of the noise influence. Fluctuations of the parameters λ_2 and λ_3 also decrease, and the parameters themselves take steady values starting with the sampling of length ~ 150 points. Applying this procedure to other data of the calm state of the Earth, we find the same behavior of noise parameters λ_i, Λ_i and detect the minimal interval of 150 points again. So, we choose the interval of such length as being optimal for accumulation of local statistics [10].

IV. DEFINITION OF RELAXATION PARAMETERS $\lambda_i(t)$ AND $\Lambda_j(t)$ FOR EARTHQUAKES AND TECHNOGENIC EXPLOSIONS DATA

It is well known that modern seismic devices allow one to derive different quantitative and qualitative data about the seismic state of the Earth. In this work we analyse three various weak local earthquakes (EQ's) in Jordan (1998) [EQ(1), EQ(2), EQ(3)], one strong earthquake in Turkey (summer, 1999) [EQ(T)] and three local underground technogenic explosions (TE) [TE(1), TE(2), TE(3)] with the length of registration from 10000 to 25000 points. In case of strong EQ its seismogram contains 65000 registered points. All these experimental data were courteously given by the Laboratory of Geophysics and Seismology (Amman, Jordan). All data correspond to transverse seismic displacements. The real temporal step of digitization τ between the registered points of seismic activity has the following values, viz, $\tau = 0.02s$ for the EQ(T), and $\tau = 0.01s$ for all others cases. We defined such characteristics as the maximal amplitude of signal fluctuations before "event" a_1, the maximal amplitude of signal fluctuations during "event" a_2, EQ (TE) power a_2/a_1, time interval till EQ (TE) T_0, continuance of "event" T_l and finally the total time of signal recording T_{total} directly from seismic data. These quantities are presented in Table I. They give a clear notion about the duration and power of investigated phenomena. One can see from Table I, approximately $4500-5700$ points are accounted for the visible part a wavelet. This number of the recorded points allows one to execute analysis by means of the local parameters $\lambda_i(t)$ and $\Lambda_j(t)$.

The procedure of calculation of time-dependence for $\lambda_i(t)$ and $\Lambda_j(t)$ was based on the following operations. An interval with $M \sim 150$ points is taken, and noisy parameters λ_i, Λ_j are calculated for it with the help of Eqs. (9), (10). Then the operation of "stepwise shift to the right" at the interval of the fixed length M is executed, and parameters

Table I: Some characteristics of Technogenic Explosions [TE(1), TE(2), TE(3)] and Earthquakes [EQ(1), EQ(2), EQ(3), EQ(T)] obtained from seismic data: a_1 is the maximal amplitude of signal oscillations before the "event", a_2 is the maximal amplitude of signal oscillations at the time of the "event", T_0 is the time from the beginning of the signal registration to the beginning the "event", T_l is the "event" duration, $T_{total} = N$ is the total time of signal recording.

	a_1	a_2	a_2/a_1	T_0	T_l	T_{total}
TE(1)	$1.03 \cdot 10^{-3}$	$8.24 \cdot 10^{-3}$	8	4091	4500	12500
TE(2)	$1.5 \cdot 10^{-3}$	$18.5 \cdot 10^{-3}$	12.3(3)	3538	4500	10000
TE(3)	$1.18 \cdot 10^{-3}$	$56.5 \cdot 10^{-3}$	47.88	4091	4850	15000
EQ(1)	$0.74 \cdot 10^{-3}$	$7.1 \cdot 10^{-3}$	9.59	5682	4500	15000
EQ(2)	$0.94 \cdot 10^{-3}$	$6.1 \cdot 10^{-3}$	6.49	12308	5770	25000
EQ(3)	$0.59 \cdot 10^{-3}$	$14.1 \cdot 10^{-3}$	23.9	3182	5700	12500
EQ(T)	$9 \cdot 10^{-3}$	20	$2.2(2) \cdot 10^3$	≈ 13500	-	65000

are computed again. These actions are executed, while the initial sampling $X(t)$ will not be finished. As a result we obtain the following dependencies $\{\lambda_i(T, T+M\tau),\ \ \lambda_i(T+\tau, T+(M+1)\tau), ..., \lambda_i(T+(N-M-1)\tau, T+(N-1)\tau)\}$ and $\{\Lambda_i(T, T+M\tau),\ \ \Lambda_i(T+\tau, T+(M+1)\tau), ..., \Lambda_i(T+(N-M-1)\tau, T+(N-1)\tau)\}$. If the character of the noise in the investigated data change, some signal will appear or disappear, and it will be directly reflected in the behavior of the relaxation characteristics.

The results of the above described procedure for the case of EQ and TE data are shown in Figs. 2, 3 and Table II. However, in order to check up the optimized length of the local interval, we calculated local parameters $\lambda_i(t)$ and $\Lambda_j(t)$ $(i = 1, 2, 3)$, $(j = 1, 2)$ at the local sampling with the length $M = 100, 200, 250, 300, 350$ and 400 points. It turned out that a large number of various noises in the behavior of $\lambda_i(t)$ and $\Lambda_j(t)$ are superimposed on the carrying trajectory at $M = 100$ points (parameters has a gross errors). The line-shapes of $\lambda_i(t)$ and $\Lambda_j(t)$ practically cease to change at the sampling the length $M = 150$ and more, $M = 200, 250, ...$ Once again it is evidence that the local interval with the length $M = 150$ points is optimal for the analysis of strong, weak EQ's and TE's.

The detailed analysis of the received results allows one to reveal the following features.

Weak EQ's and local underground TE's (for TE(3) and EQ(3), see Figs. 2, 3):

1. All the parameters $\lambda_i(t)$ take only negative values ($\lambda_i(t) < 0$), whereas the noisy parameters $\Lambda_j(t)$ can be both positive and negative.

2. *Noisy parameter* $\lambda_1(t)$. The absolute magnitude $|\lambda_1|$ increases sharply in its amplitude by a factor approximately equal to $4 - 13.3$ during EQ (various for different EQ's), and then it returns to its initial state. the restoration time T_{λ_1} and the duration of "event" T_l are approximately equal for weak EQ, i. e. $T_{\lambda_1} \approx T_l$.

The absolute magnitude $|\lambda_1|$ also exhibits an abrupt rise $\sim 2.2 - 3.5$ times higher for TE's. However, it returns quickly to its normal level. The restoration time $\lambda_1(t)$ for TE is less than the duration of the "event" approximately by a factor of $2.5 - 3$.

3. *Noisy parameter* $\lambda_2(t)$. The parameter $\lambda_2(t)$ responds to the beginning of the "event" by an abrupt rise of its value. It increases in amplitude both for TE's and for EQ's. The character of the noise changes during the "event". The parameter responds to the power of the "event".

4. *Noisy parameter* $\lambda_3(t)$. This parameter always fluctuates near its numerical value -1, and has an abrupt rise at the beginning the "event" in the form of separated spikes (see Figs. 3 and 4). The parameter keenly responds to the noise changes.

5. *Noisy parameter* $\Lambda_1(t)$. It fluctuates near zero before and after the "event" changing its sign at this time. The parameter increases sharply at the "event" and *always* **(!)** retains positive. Then it decays smoothly. Restoration time T_{Λ_1} and the duration of the "event" T_l are approximately the same both for the EQ and TE. This parameter is very sensitive to the changes of the noise character. For example, the data analysis of the EQ(2) shows that the separate burst of the amplitude values of $\Lambda_1(t)$ appears for ≈ 4000 points up to the beginning of the EQ, although such indicator was not visually observed in the initial seismic data [17]. It may be the evidence of high prognostic property of this parameter for EQ's forecasting.

6. *Noisy parameter* $\Lambda_2(t)$. Noise changes of the parameter $\Lambda_2(t)$ are observed during the "event" both during EQ's and TE's. From Figs. 2 and 3 we can see that $\Lambda_2(t)$ has a distinctive negative depression during the "event".

Strong EQ's (for EQ(T) see Fig. 4) [18]:

All parameters react keenly to the appearance of the signal in case of a strong EQ. Let us consider the behavior of the local parameters in this case in detail.

7. *Noisy parameter* $\lambda_1(t)$. The parameter demonstrates the presence of noise and takes negative values before the "event" [Region *I* in Fig. 4 f)]. As the "event" approaches, the amplitude of fluctuations decreases, and oscillations turn into negligible fluctuations near zero value for II, III and IV Regions (for more detail, see Fig. 4).

8. *Noisy parameter* $\lambda_2(t)$. A noise is also observed in the behavior of this parameter before the "event", and this parameter takes only negative values. However, its values decrease sharply in absolute magnitude and begin to take values near zero with the appearance of an EQ signal (Fig. 4 b).

9. *Noisy parameter* $\lambda_3(t)$. This parameter takes only negative values at all times t. The negligible noise appears before the "event'. As the "event" approaches the noise increases in amplitude by the factor $3-5$. The amplitude of oscillations begins to decrease in Region IV (see Fig. 4 c).

10. *Noisy parameters* $\Lambda_1(t)$ *and* $\Lambda_2(t)$. Parameters fluctuate, taking positive values mainly before the "event" (Region I). With the beginning of "event" values of parameter increase greatly in absolute magnitude approaching zero. At the beginning of Region II both parameters change their sign from positive to negative. Then for Regions II, III and IV we see only right line with $\Lambda_1(t), \Lambda_2(t) \to 0$ on the scales of Figs. 4 d) and e). However, the parameter $\Lambda_2(t)$ has faintly visible fluctuations for Region IV characterizing the final EQ phase [see Figs. 4 d) and e)].

So, all parameters are very sensitive to the approach of a strong EQ. A sharp change in their behavior is appreciable before strong fluctuations of the Earth's surface far off 10000 points (~ 3.5 minutes).

Table II: Characteristics of local parameters $\lambda_1(t)$ and $\Lambda_1(t)$: T_{λ_1} is the parameter $\lambda_1(t)$ relaxation time, T_{Λ_1} is the relaxation time of $\Lambda_1(t)$, τ_λ is the relaxation time for exponential attenuation of $\lambda_1(t)$, τ_Λ is the relaxation time for exponential attenuation of $\Lambda_1(t)$.

	λ_0 (units of τ^{-1})	$\triangle\lambda$ (units of τ^{-1})	T_λ	τ_λ	Λ_0 (units of τ^{-2})	$\triangle\Lambda$ (units of τ^{-2})	T_Λ	τ_Λ	$\Delta\lambda/\lambda_0$	$\Delta\Lambda/\Lambda_0$	T_l/T_λ
TE(1)	-0.13	-0.425	1800	90	0.02	0.28	4500	45	3.27	14	2.5
TE(2)	-0.15	-0.34	1440	100	0.02	0.28	4032	55	2.26(6)	14	3.125
TE(3)	-0.15	-0.33	1870	80	0.002	0.42	4850	45	2.2	210	2.59
EQ(1)	-0.045	-0.6	4500	170	0.005	0.43	4500	110	13.3(3)	86	1
EQ(2)	-0.1	-0.5	5770	170	0.001	0.35	5770	130	5	350	1
EQ(3)	-0.12	-0.47	5700	210	0.01	0.35	5700	180	3.91(6)	35	1

V. SIMPLE EXPONENTIAL MODEL FOR TIME LOCAL BEHAVIOR OF NOISY RELAXATION PARAMETERS $\lambda_1(t)$ AND $\Lambda_1(t)$

As can be seen from Figs. $2-4$ all relaxation parameters are very sensitive to the beginning of EQ and TE. The behavior of parameters $\lambda_1(t)$ and $\Lambda_1(t)$ in weak local EQ's and local TE's is of great interest. These parameters are initial in our calculations [see Eqs. (9), (10)]. The analysis has shown that parameters oscillate near average values λ_0 and Λ_0, correspondingly, before and after the oscillations visible on seismograms. The results of the behavior of these parameters for TE(3) and EQ(3) are presented in Figs. 2 a)-d) and Fig. 3 a)- d). However, a sudden rise by factors $\Delta\lambda_0$ and $\Delta\Lambda_0$ is always observed in the behavior of these parameters at the enhancement or the appearance of the signal (it is seen at the beginning of the EQ or the TE in seismogram data). Furthermore, the continuous attenuation occurs. Over all this time these parameters have a well-defined pronounced trend. Such behavior of $\lambda_1(t)$ and $\Lambda_1(t)$ give us a possibility of modelling the time dependence of these parameters by some simple mathematical functions. The fitting procedure showed that the time behavior of these parameters can be well approximated by the following simple time dependencies:

$$\lambda_1(t) = \lambda_0 + \Delta\lambda \cdot \exp\left\{-\frac{t-T_0}{T_\lambda}\right\} \cdot H(t-T_0), \tag{46}$$

$$\Lambda_1(t) = \Lambda_0 + \Delta\Lambda \cdot \exp\left\{-\frac{t-T_0}{T_\Lambda}\right\} \cdot H(t-T_0), \tag{47}$$

where $H(t)$ is the Heaviside function, T_λ and T_Λ are the relaxation times of $\lambda_1(t)$ and $\Lambda_1(t)$, correspondingly. The time T_0 is the same in Eqs. (46) and (47) for parameters $\lambda_1(t)$ and $\Lambda_1(t)$ (see Table I). The numerical values of the variables, included in Eqs. (46), (47) were defined for EQ's and TE's by comparison of localization results with these equations (see Fig. 5). Numerical values of parameters are presented in Table II.

So, it proved that the restoration of these parameters to their steady values occurs according to the *exponential* law. As can be seen from Fig. 5 this description best suits for $\lambda_1(t)$ and $\Lambda_1(t)$ of weak EQ's. The foregoing estimations strengthen fully our resume **2** and **5** of Section IV.

The results presented in the last three columns of Table II might be of interest for readers. As the quantities $\triangle\lambda_0/\lambda_0$ and $\triangle\Lambda_0/\Lambda_0$ show the rise of value of the corresponding parameter at TE and EQ. Finally, the ratio between the "event" duration T_l and the relaxation time T_λ discovers a remarkable distinction between TE's and weak EQ's.

VI. CONCLUSION

In this work universal method for investigating non-stationary, unsteady and non-Markov random processes in discrete systems is suggested. This universality is achieved by combining the opportunities of microscopic, mesoscopic and macroscopic descriptions for complex systems. This method allows one to find and to analyze fast, slow and super-slow processes. To investigate super-slow processes we propose to use the model of a "quasi-Brownian particle". The wavelet-transformation of the initial time series can be used for this purpose. This method helps to analyze and differentiate similar signals of different origin. Theoretical investigations have been realized by means of two methods supplementing each other: the statistical theory of discrete non-Markov stochastic processes [2] and the local noisy parameters. The application of the last method gives a possibility to study non-stationary and unsteady processes with alternation and superimposition of different modes. The correlation between the time scales characteristic of different modes may be different. However, at accurate realization the localization procedure allows one to separate the noise and the signals [1, 2], and to carry out their quantitative and informative analysis.

Another important advantage of this method is the possibility to operate it in "real time" regime, i.e. it can be put into practice immediately at getting the data, that is of great practical value.

The developed approach has been tested for strong and weak EQ's data and nuclear underground TE's. As a result we have obtained of the following results.

The time-behavior of the local relaxation parameters can be described by simple model relaxation functions. The temporal relaxation of parameters $\lambda_1(t)$ and $\Lambda_1(t)$ in weak EQ's and TE's after the beginning of the "event" occurs according to the exponential law. However, the restoration and duration of the events are practically the same in case of weak EQ's. The restoration time of parameters $\lambda_1(t)$ and $\Lambda_1(t)$ in the case of TE's differs noticeably from the duration of the event. So, this approach can be useful in recognition these two different seismic phenomena. From the analysis of strong EQ data one can see that the behavior of all parameters changes greatly long before the EQ. For example, such change for the EQ(T) presented here occurs ~ 3.5 minutes before the main event. This change of $\lambda_i(t)$ and $\Lambda_i(t)$ obtained for strong EQ's opens the possibility for a more accurate registration of the beginning of changes of the parameters before the visual wavelet and real EQ's.

We are sure that the suggested method can be very useful for the study of a wide class of random discrete processes in real complex systems of live and of lifeless things: in cardiology, physiology, neurophysiology, biophysics of membranes and seismology.

VII. ACKNOWLEDGMENTS

We gratefully acknowledge Prof. Raoul Nigmatullin for stimulating discussion and Dr.L.O.Svirina for technical assistance. This work supported in part by Russian Fund for Basic Research (Grant No 02-02-16146), Ministry of Education of RF (Grant No A03-2.9-336, No 02-3.1-538) and RHSF (No 03-06-00218a).

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Appendix A: THREE FORMS OF THE QUASI-OPERATOR $\hat{\mathcal{L}}$

Equations of motion of a random variable x with the use of the derivative of the three different forms [29] are represented here.

Using evolution operator, we can write the equation of motion for a discrete case as following

$$dx(t)/dt = i\hat{\mathcal{L}}x(t). \tag{A1}$$

However, there is a possibility of application of the time derivative $d/dt \to \triangle/\triangle t$ in three different forms:

1. *"Right" derivative (with decurrent difference in numerator)*

$$\frac{\triangle x(t)}{\triangle t} = \frac{x(t+\tau) - x(t)}{\tau} = \frac{1}{\tau} U(t+\tau, t)x(t) \tag{A2}$$

with Liouville's quasioperator of the following form:

$$\hat{\mathcal{L}}(t,\tau) = -\frac{i}{\tau}[U(t+\tau,t) - 1]; \tag{A3}$$

2. *"Left" derivative (with ascending difference in numerator)*

$$\frac{\triangle x(t)}{\triangle t} = \frac{x(t) - x(t-\tau)}{\tau} = \frac{x(t) - U^{-1}(t,t-\tau)x(t)}{\tau} = \frac{1}{\tau}[1 - U^{-1}(t,t-\tau)]x(t) \tag{A4}$$

with the Liouvillian of the next form:

$$\hat{\mathcal{L}}(t,\tau) = -\frac{i}{\tau}[1 - U^{-1}(t,t-\tau)]; \tag{A5}$$

3. *"Central" derivative (with central difference in numerator)*

$$\begin{aligned} \frac{\triangle x(t)}{\triangle t} &= \frac{x(t+\tau) - x(t-\tau)}{2\tau} = \frac{x(t+\tau) - x(t)}{2\tau} - \frac{x(t) - x(t-\tau)}{2\tau} \\ &= \frac{1}{2\tau}[U(t+\tau,t) - U^{-1}(t,t-\tau)]x(t). \end{aligned} \tag{A6}$$

Then the quasioperator $\hat{\mathcal{L}}$ takes the following form:

$$\hat{\mathcal{L}}(t,\tau) = -\frac{i}{2\tau}[U(t+\tau,t) - U^{-1}(t,t-\tau)]. \tag{A7}$$

In the calculations and the analysis presented in this work we have used the derivative of the first form [see Eqs. (A2), (A3)].

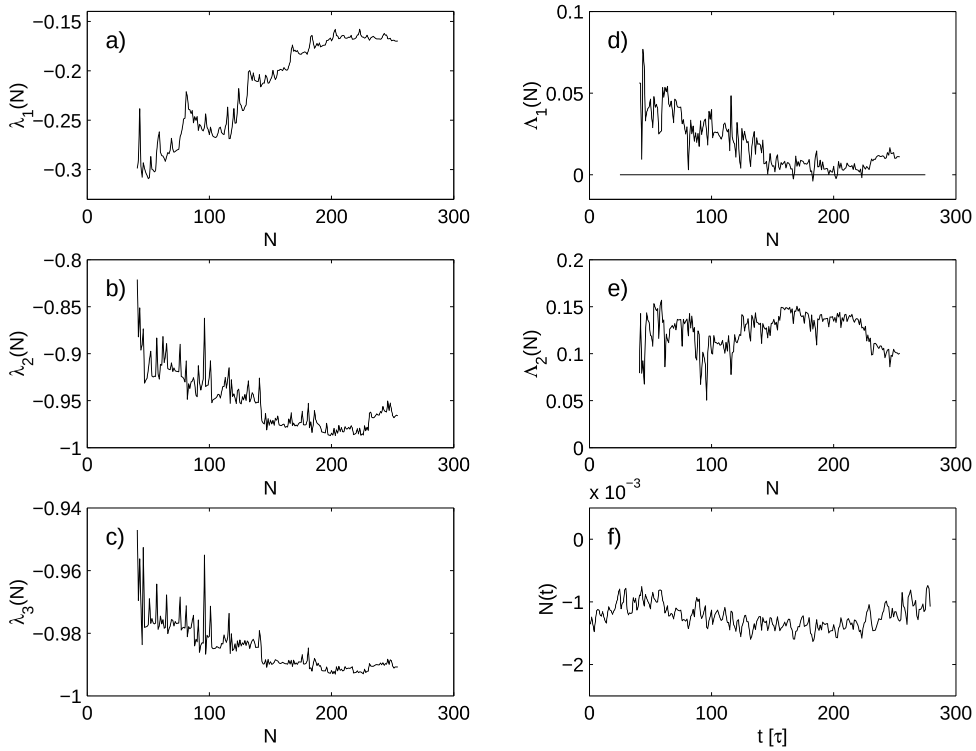
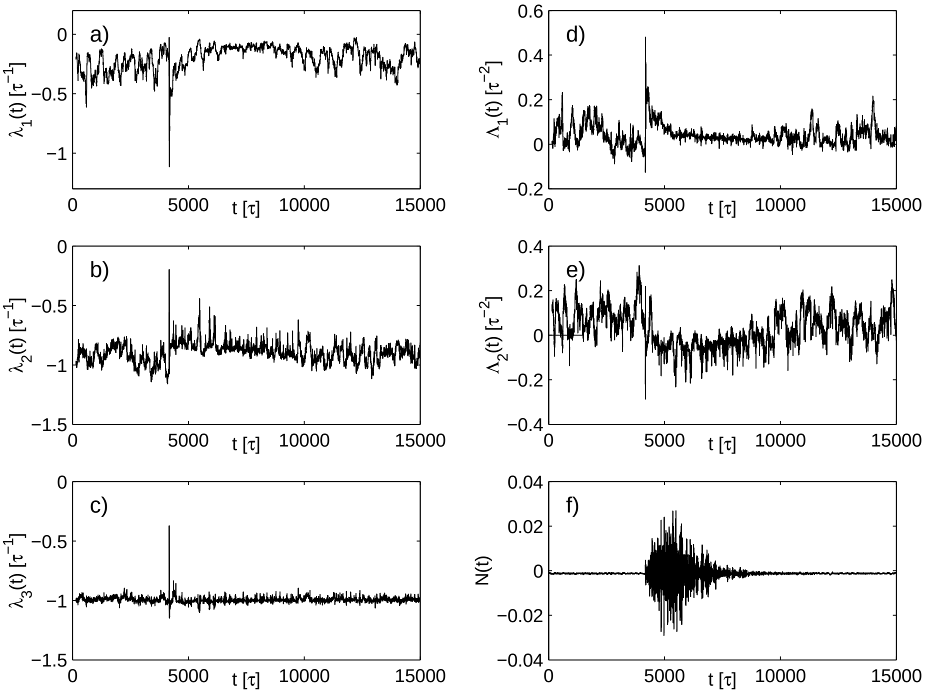


Figure 1: The definition of the optimal size for a local "working window". Calculations were made for the data of a steady state of the Earth. As a result we find that size $N \sim 150$ points is more optimal. The relative stability of all parameters is observed. Inset f) shows seismic signals for the Earth's steady state (before underground TE).



F

Figure 2: The calculated time behavior of noisy relaxation parameters λ_i and Λ_i for the technogenic explosion TE(3) [figures a)-e)], the signal of which is presented at inset f).

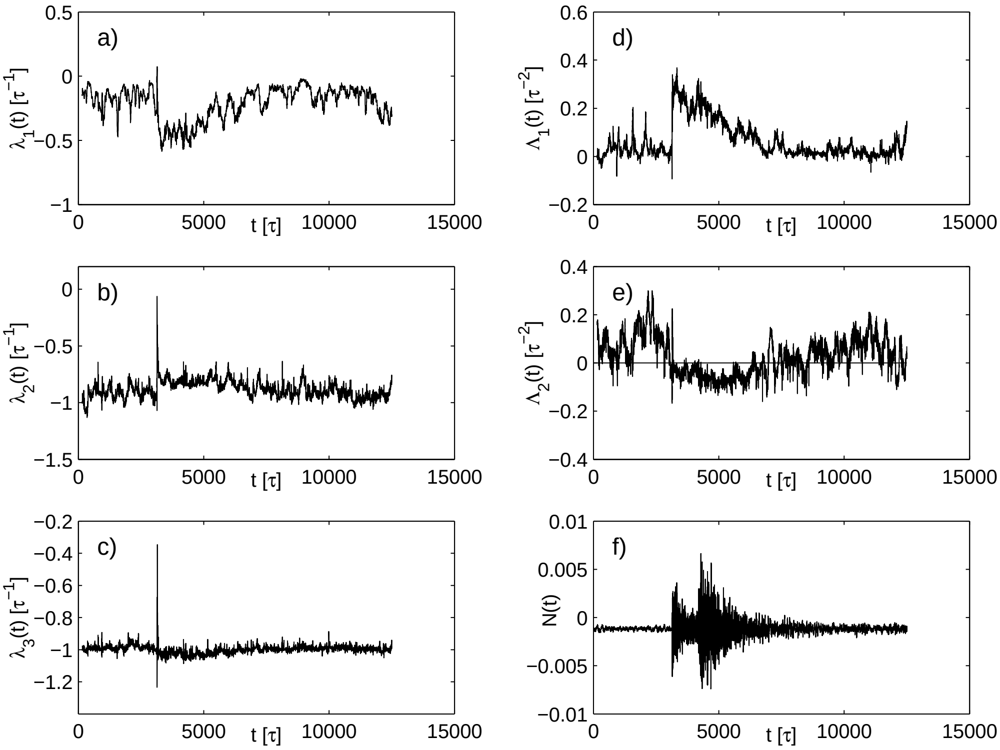


Figure 3: The time behavior of the noise relaxation parameters λ_i and Λ_i for the weak EQ(3) [see figures a)-e)], the signal of which is also presented at inset f).

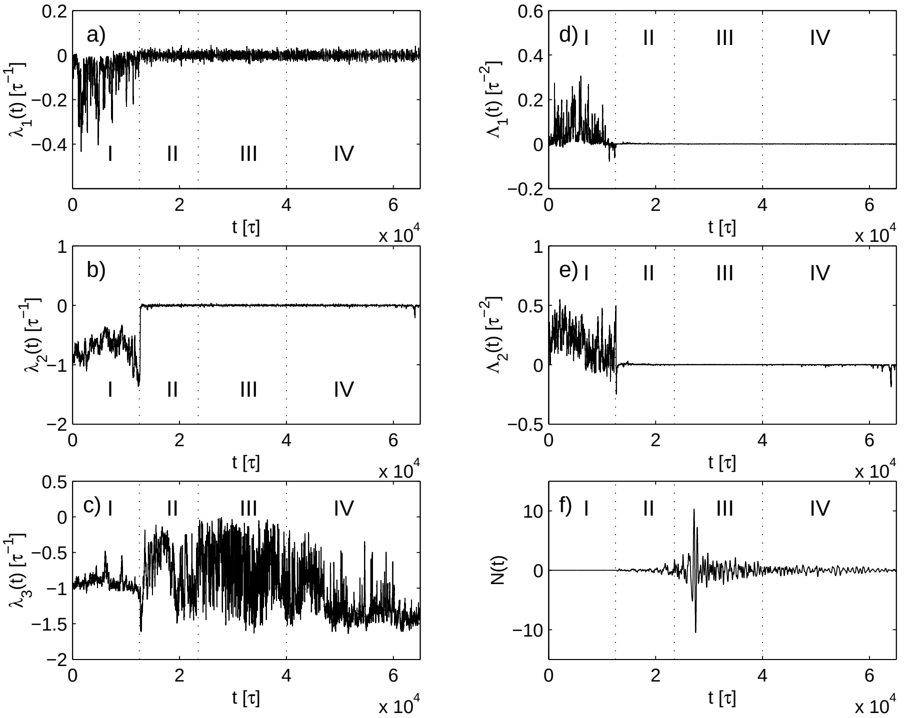


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Figure 4: The time behavior of the noisy relaxation parameters λ_i and Λ_i [figures a)-e)] for the strong EQ(T), the signal of which is presented in figure f). The following regions: I - a calm state of the Earth's core, II - the state before the earthquake, III - the state during the earthquake and, finally, IV - the state after the event are divided by the vertical dotted lines.

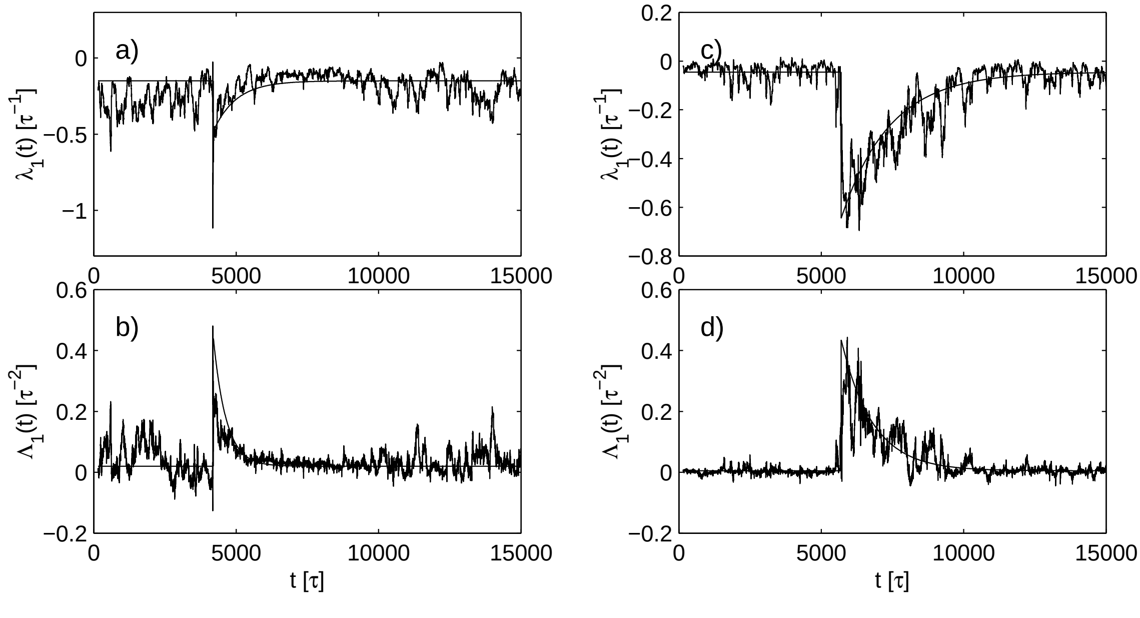

Figure 5: The time behavior of the first two noisy relaxation parameters λ_1, Λ_1 for TE(3) [figures a) and b)] and for the weak EQ(1) [figures c) and d)]. Solid lines show fitted functions (46) and (47) with corresponding parameters, presented in Table II. One can note the typical exponential restoration of the parameters with the beginning of the event.